

Unit - Structure of Atom

Topics - Quantum Mechanical Model

To overcome the limitation of Bohr's model attempts were made to search a more developed suitable model of atom.

There are two important concepts which led to development of such a model & they are

- (1) The Dual-behaviour of Matter
- (2) Heisenberg's Uncertainty Principle.

The Dual-Behaviour of Matter

De-broglie Concept:- Like light (exhibits both wave & particle properties) all microscopic material particle such as electrons, protons, atoms, molecules etc. have also dual-behaviour.

According to de-broglie,

All material particles in motion possess wave characteristics. Its wave-length associated with a particle mass (m) with velocity (v) is related as

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \left| \begin{array}{l} p = \text{momentum} \\ h = \text{Planck's constant} \end{array} \right.$$

The wave associated with material-particle are called matter/de-broglie waves.

Derivation of de-broglie-Relation

It can be obtained by combining the mass-energy relationship which was proposed by Max-Planck & Einstein.

According to Max-Planck's,

$$E \propto \nu$$

$$\text{or, } E = h\nu$$

$E = \text{Energy}$
 $\nu = \text{frequency of wave}$
 $h = \text{Planck's Constant}$

According to Einstein,

$$E = mc^2$$

$m = \text{mass of particle}$
 $c = \text{velocity of light}$

After combining both

$$h\nu = mc^2$$

$$\text{or, } h \cdot \frac{c}{\lambda} = mc^2$$

$$\therefore \lambda = \frac{h}{mc}$$

$$\text{or, } \lambda \propto \frac{1}{p} \quad \left\{ \lambda = \frac{h}{mv} \right\}$$

$v = \text{velocity of that particle}$
 $p = \text{momentum} = mv$

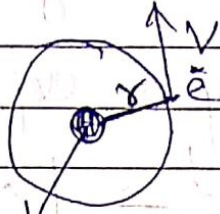
The above equation is known as de-broglie relation.

Significance of de-broglie Relationship

It is applicable to all material objects but it is significant only for microscopic bodies not upon macro (big) particles. The reason is that like particle waves do not occupy a well-defined position in space and are delocalized. Thus it has no significance for big particles ~~as their~~ because wave-length of wave associated with them is very very small to be detected.

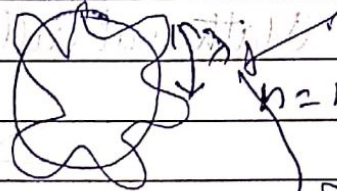
Some aspects from de-broglie relationship.

- (1) Derivation of Bohr's postulates of quantisation of angular momentum from de-broglie.



Nucleus

$$\text{Circumference} = 2\pi r$$



$n = \text{number of waves}$

wave. length $\lambda = \text{wave-length}$

$$\text{Circumference} = n\lambda$$

For a closed-orbit, Circumference should be integral-multiple of wave-length.

$$2\pi r = n\lambda$$

$$2\pi r = n \cdot \frac{h}{mv}$$

Thus:

$$mvr = \frac{nh}{2\pi}$$

Thus angular-momentum (mvr) is quantised for closed-orbit.

- (2) de-broglie wave-length related with kinetic-energy (K.E) of electron.

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m \cdot \sqrt{\frac{2 \cdot KE}{m}}}$$

$$\lambda = \frac{h}{\sqrt{2 \cdot KE \cdot m}}$$

$$K.E = \frac{1}{2} mv^2$$

$$\therefore v = \sqrt{\frac{2 \cdot KE}{m}}$$

- (3) Calculation of number of waves in an orbit.

Number of waves in an orbit

$$= \frac{\text{Circumference of orbit}}{\text{wave-length}}$$

$$= \frac{2\pi r}{\lambda} = \frac{2\pi r}{h} \times mv = \frac{mvr \times 2\pi}{h} = \frac{nh}{2\pi} \times \frac{2\pi}{h} = n$$

2. Heisenberg's Uncertainty Principle

Statement:- It is impossible to determine both position & velocity (or momentum) of a microscopic particle simultaneously with absolute accuracy.

Mathematically, it may be expressed as

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \times m \Delta v \geq \frac{h}{4\pi}$$

Also expressed as

$$\Delta E \times \Delta t \geq \frac{h}{4\pi}$$

Δx = uncertainty in position
 Δp = uncertainty in momentum
 ΔE = uncertainty in energy
 Δt = uncertainty in time

The product of Δx & Δp can be either greater or equal to $\frac{h}{4\pi}$ but never less than $\frac{h}{4\pi}$

Simplification of Uncertainty Principle

Bohr proposed the concept of definite path or trajectories of electron (both location & velocity must known at each moment) in an atom is completely ruled out by Heisenberg.

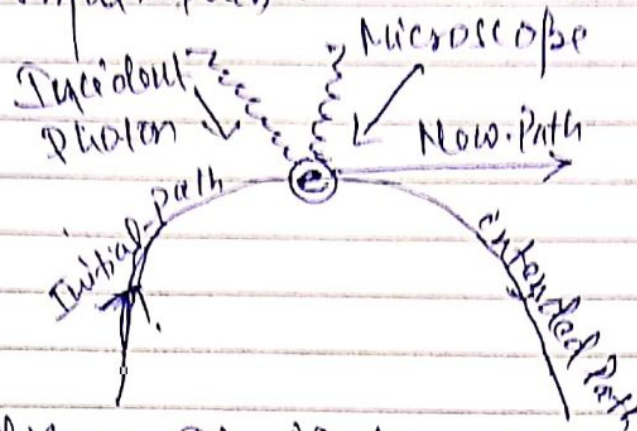
Heisenberg replace the concept of definite orbit by concept of probability.

Physical Concept of Uncertainty Principle

It is not possible to determine simultaneously the exact position and momentum of the electron of any other microscopic.

Now even in case of microscopic particle like electron, the interaction of the

Striking photon of light with particle cause appreciable displacement of the particle from normal path.



The collision of photon with electron sends the electron in an unpredictable path.

Significance of uncertainty principle in our

In our daily life it has no significance. Because we always think about large objects (observe with naked eye) without altering their motion.

we can determine their position & velocity accurately.

Why - electron do not present in the nucleus?

As we know that the size of nucleus of the order of 10^{-15} m. If it present in nucleus then their uncertainty in position has to be 10^{-15} m.

From Heisenberg relation.

$$\Delta x \times \Delta p = \frac{h}{4\pi}$$

$$\therefore \Delta v = \frac{h}{4\pi m \times \Delta x} = \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 9.108 \times 10^{-31} \times 10^{-15}} = 5.77 \times 10^{10} \text{ m/s.}$$

The Δv is very larger with respect to light velocity. Thus it cannot exist in the atomic nucleus.